

# Spectra of a weakly driven two-level atom in an off-resonant squeezed vacuum

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**Abstract.** We have derived analytical expressions for the absorption-dispersion spectra as well as the steady state fluorescence spectrum of a weakly excited single 2-level atom in a broadband squeezed vacuum. The atomic transition frequency and the central frequency of both the squeezed vacuum field and the driving field are assumed different. The absorption-dispersion and resonance fluorescence spectra are found to be very much dependent on the atomic as well as squeezed vacuum detuning and can lead to the situation of negative group velocities. Also, the resonance fluorescence spectrum is sensitive to the relative phase of squeezed vacuum and the driving field in the presence of finite detuning of the squeezed vacuum field.

**PACS.** 32.80.-t Photon interactions with atoms – 42.50.Ct Quantum description of interaction of light and matter; related experiments – 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements

## 1 Introduction and model equations

Generation of squeezed light in the laboratory [1] (also see [2]) has been successfully demonstrated over the past many years and many possible and interesting applications of such a light source have been discussed. It was Gardiner [3] who first considered the radiative decay of a two-level atom in a broadband squeezed bath and pointed out that one of the polarization quadrature of the atom decays at a reduced rate when compared to a normal bath. As a consequence of this study, the possibility of obtaining sub-natural linewidth in resonance fluorescence spectrum [4] as well as in weak field absorption spectrum [5] of a two-level atom embedded in a broadband squeezed vacuum has been demonstrated. Further, recent investigation [6] shows that the spectra of a probe beam probing a two-level atomic system damped by a broadband squeezed vacuum and driven by a weak laser field exhibits finite refractive index accompanied with zero absorption when the probe beam is at resonance with the laser frequency. This resonant finite refractive index-zero absorption effect [6] is due to the coherent population oscillations [7,8] — which is significantly enhanced by the presence of the squeezed vacuum field induced by the driving laser and probe fields beating together at their difference frequency. Note that in the normal vacuum the possibility of exhibiting high refractive index with zero absorption has been demonstrated for a system of single atom with lambda structure and

prepared in a phase coherent state by Scully [9] and for other multilevel atomic systems by Scully *et al.* [10] — also see [11] for experimental observation and references in [6].

In the studies [5,6], it was assumed that the frequency of the driving laser field is equal to the carrier frequency of the squeezed vacuum field. Here we examine how the off-resonant squeezed vacuum field affects the spectral features of the absorption and the dispersion spectra pertaining to the production of unusual group velocities. Also, we examine the resonance fluorescence (RF) spectrum for the same system and show the sensitivity of the RF spectrum with weak laser field to both the detuning and the relative phase of the off-resonant squeezed vacuum field.

The model Bloch equations governing the dynamics as well as spectral properties for a single 2-level atom of transition frequency  $\omega_0$  and driven by a coherent field of frequency  $\omega_L$  in the presence of a broadband squeezed vacuum of central frequency  $\omega_s$ , in a rotating frame at  $\omega_L$ , are given by [12–14]

$$\begin{aligned}\langle \dot{S}_+ \rangle &= -[(\gamma/2)(2N+1) + i\Delta]\langle S_+ \rangle \\ &\quad - \gamma M \exp(2i\delta t)\langle S_- \rangle - 2i\Omega_0\langle S_z \rangle, \\ &= \langle \dot{S}_- \rangle^*, \\ \langle \dot{S}_z \rangle &= -\gamma/2 - \gamma(2N+1)\langle S_z \rangle + i\Omega_0(\langle S_- \rangle - \langle S_+ \rangle),\end{aligned}\quad (1)$$

in which  $\gamma$  is the Einstein  $A$  coefficient,  $\Omega_0$  is the Rabi frequency associated with the coherent field,  $N$  and  $M(=|M|e^{i\Phi})$  are the squeezed vacuum parameters ( $\Phi$  is

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the relative phase of the squeezed vacuum field with respect to the driving field) with  $|M| \leq (N(N+1))^{1/2}$ ,  $\Delta = \omega_L - \omega_0$  is the atomic detuning and  $\delta = \omega_L - \omega_s$  is the squeezed vacuum detuning. The atomic operators  $S_{\pm,z}$  obey the following commutator relations

$$[S_z, S_{\pm}] = \pm S_{\pm}, \quad [S_+, S_-] = 2S_z. \quad (2)$$

In the following two sections, Sections 2 and 3, we calculate in the weak coherent field case the absorption-dispersion spectra and the RF spectra. A summary is given in Section 4.

## 2 The absorption and dispersion spectra

The absorption and dispersion spectra for a system of two-level atoms can be easily calculated by employing the Kramers-Kronig dispersion relations. With this relation the complex susceptibility and hence the complex refractive index of a weak field probing the atomic system can be determined. After the system of a driven two-level atom interacting with a squeezed vacuum bath attains equilibrium, it is probed by a weak field of amplitude  $\epsilon_P$  and frequency  $\nu$ . The linear susceptibility  $\chi(\nu)$  of the probe beam is given in terms of the Fourier transform of the expectation values of the two-time commutator of the atomic operators  $S_-$  and  $S_+$  [15]

$$\chi(\nu) = |\boldsymbol{\mu} \cdot \boldsymbol{\epsilon}_P|^2 N_a \int_0^{\infty} \text{Lim}_{t \rightarrow \infty} \langle [S_-(t+\tau), S_+(t)] \rangle e^{i(\nu - \omega_L)\tau} d\tau, \quad (3)$$

where  $\boldsymbol{\mu}$  is the transition electric dipole moment of the atom and  $N_a$  is the atomic density. The absorption and dispersion spectra are determined by the real and the imaginary parts of the susceptibility  $\chi(\nu)$  normalized to  $|\boldsymbol{\mu} \cdot \boldsymbol{\epsilon}_P|^2 N_a$ :

$$\begin{aligned} G_a(\nu) &= \text{Re}[\chi(s)]_{s=i(\nu - \omega_L)}, \\ G_d(\nu) &= \text{Im}[\chi(s)]_{s=i(\nu - \omega_L)}, \end{aligned} \quad (4)$$

where  $\chi(s)$  is the Laplace transform (on  $\tau$ ) of the commutator  $\text{Lim}_{t \rightarrow \infty} \langle [S_-(t+\tau), S_+(t)] \rangle$  and can be calculated by making use of the following expression according to equations (1) in the weak field case to  $O(\Omega_0^2)$ ,

$$\chi(s) = \frac{2}{\rho(s - i\delta)} [i\Omega_0 B_z(s - i\delta) F_-(\infty) + B_+(s - i\delta)(F_z(\infty) - 1/2)], \quad (5)$$

with

$$\begin{aligned} 2\Gamma &= \gamma(1 + 2N), \quad M = |M|e^{i\Phi} \\ \rho(s) &= (s + \lambda_1)(s + \lambda_1^*) - \gamma^2 |M|^2, \\ &= (s + s_1)(s + s_2), \\ s_{1,2} &= \Gamma \mp [\gamma^2 |M|^2 - (\Delta + \delta)^2]^{1/2}, \\ \lambda_1 &= \Gamma + i(\Delta + \delta), \quad |M|^2 = N(N + 1). \end{aligned} \quad (6)$$

The quantities  $B_z(s)$  and  $B_+(s)$  have the following expressions

$$\begin{aligned} B_z(s) &= (s + \lambda_1^*)a_z(s) + \gamma M a_z^*(s), \\ B_+(s) &= (s + \lambda_1^*)(1 + 2\Omega_0^2 a_+(s)) - 2\gamma M \Omega_0^2 a_-^*(s) \end{aligned} \quad (7)$$

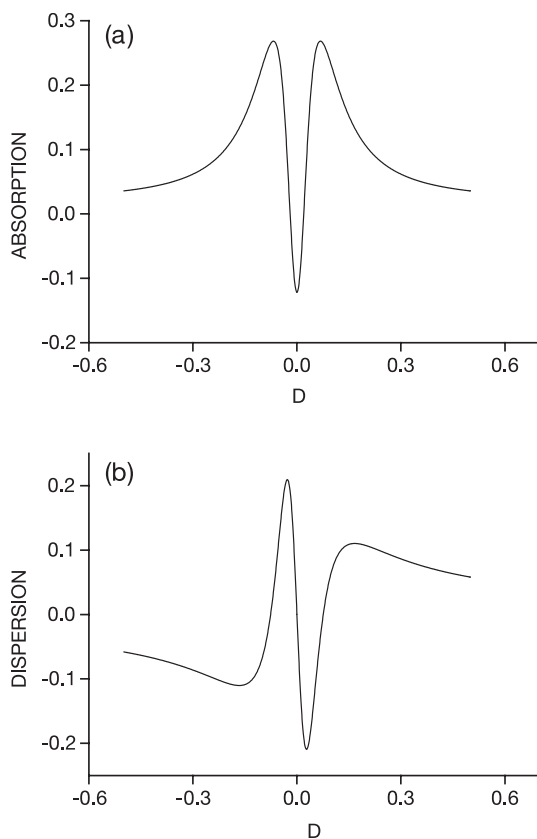
and the coefficients  $a_z(s)$ ,  $a_+(s)$  and  $a_-(s)$  are defined as

$$\begin{aligned} a_z(s) &= \frac{1}{s + 2\Gamma + i\delta}, \\ a_+(s) &= \frac{\Gamma + i(\Delta + 2\delta)}{(s_1 + i\delta)(s_2 + i\delta)(s + 2\Gamma + i\delta)} \\ &+ \frac{1}{(s_2 - s_1)} \left[ \frac{(s_1 - \lambda_1^*)}{(s + s_1)(s_2 + i\delta)} \right. \\ &\left. - \frac{(s_1 - \lambda_1^*)}{(s + s_1)(s_2 + i\delta)} \right] \\ &+ \gamma M^* \left[ \frac{1}{(s_2 - s_1)} \left( \frac{1}{(s_1 - i\delta)(s + s_2 + 2i\delta)} \right. \right. \\ &\left. \left. - \frac{1}{(s_2 - i\delta)(s + s_1 + 2i\delta)} \right) \right. \\ &\left. - \frac{1}{(s_1 - i\delta)(s_2 - i\delta)(s + 2\Gamma + i\delta)} \right], \\ a_-(s) &= \frac{-\Gamma + i(\Delta + 2\delta)}{(s_1 - i\delta)(s_2 - i\delta)(s + 2\Gamma + i\delta)} \\ &+ \frac{1}{(s_2 - s_1)} \left[ \frac{(s_2 - \lambda_1)}{(s + s_2 + 2i\delta)(s_1 - i\delta)} \right. \\ &\left. - \frac{(s_1 - \lambda_1)}{(s + s_1 + 2i\delta)(s_2 - i\delta)} \right] \\ &+ \gamma M \left[ \frac{1}{(s_2 - s_1)} \left( \frac{1}{(s_2 + i\delta)(s + s_1)} \right. \right. \\ &\left. \left. - \frac{1}{(s_1 + i\delta)(s + s_2)} \right) \right. \\ &\left. + \frac{1}{(s_1 + i\delta)(s_2 + i\delta)(s + 2\Gamma + i\delta)} \right], \end{aligned} \quad (8)$$

with

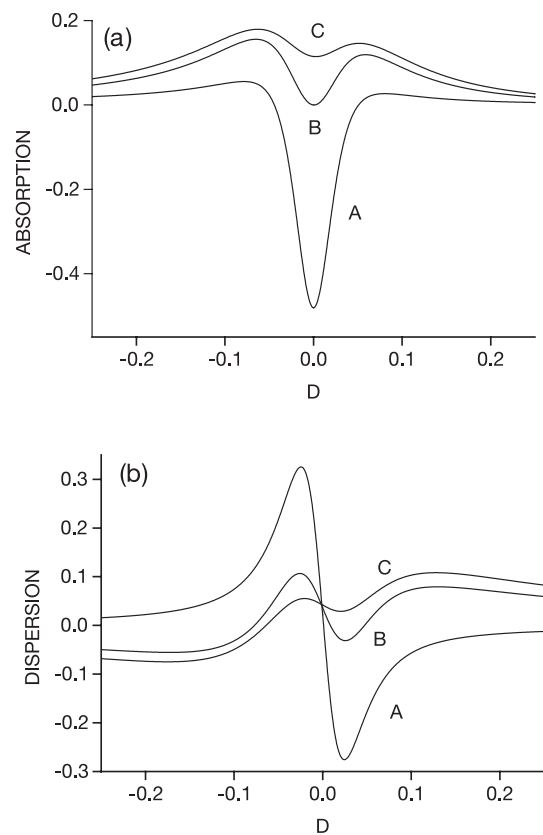
$$\begin{aligned} F_-(\infty) &= \frac{-i\Omega_0}{(1 + 2N)} \frac{(\Gamma + i(\Delta + 2\delta))}{\rho(i\delta)}, \\ F_z(\infty) &= \frac{N}{1 + 2N} \left[ 1 + \frac{\Omega_0^2}{N} \alpha \right], \\ \alpha &= \frac{\gamma^2/4 + (\Delta + 2\delta)^2}{(\gamma^2/4 + \Delta(\Delta + 2\delta))^2 + 4\Gamma^2\delta^2}. \end{aligned} \quad (9)$$

We now depict the absorption and dispersion spectra pictorially as a function of  $D = (\nu - \omega_L)/\gamma$  in Figures 1–3 for different choices of the system parameters. In Figures 1a and 1b, we keep  $N = 5.0$ ,  $\Phi = \pi$ ,  $\delta/\gamma = 0$ ,  $\Delta/\gamma = 0.0$ ,  $\Omega_0/\gamma = 0.21$  and  $|M| = \sqrt{N(N+1)}$ . Figure 1a shows the absorption spectrum under weak field excitation and for a resonant squeezed vacuum field.



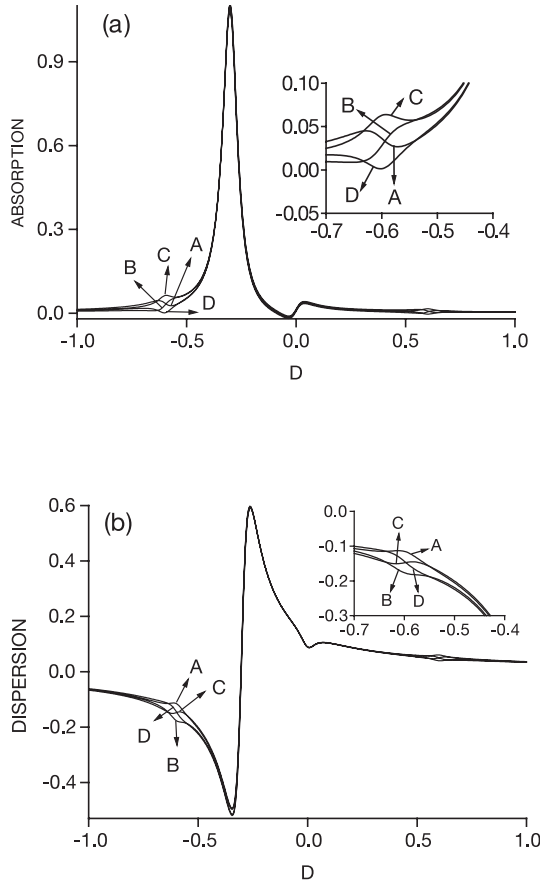
**Fig. 1.** (a) The absorption (in arbitrary unit) as a function of  $D = (\nu - \omega_L)/\gamma$  (dimensionless) for a two-level atom damped by a broadband squeezed vacuum ( $N = 5$ ,  $\Phi = 0$ ,  $\delta/\gamma = 0$ ,  $\Delta/\gamma = 0$ ,  $|M| = \sqrt{N(N+1)}$ ) and driven by a weak laser field  $\Omega_0/\gamma = 0.21$ . (b) The dispersion (in arbitrary unit) as a function of  $D = (\nu - \omega_L)/\gamma$  (dimensionless) for a two-level atom damped by a resonant broadband squeezed vacuum with data as in (a).

The absorption spectrum shows negative peak or gain for these parameters. The dispersion spectrum (Fig. 1b) shows an anomalous dispersion near  $D = 0$  region. Note that in an experimental demonstration [16] it has been shown that the optical pulses propagate twenty million times slower than the speed of light in vacuum. For this purpose an ultra cold gas of sodium atoms (cooled down to a nanoKelvin temperature) has been used and the phenomenon of quantum interference is found to be responsible for this effect [16]. The gist of this effect of slowing down can be briefly summarized as follows: in the cloud of ultra cooled Na atoms, a coupling laser beam is passing through and it is making the cloud transparent to light of a precise frequency and at the same time causing sharp variation of the refractive index. Because of this transparency a properly tuned light can pass through the cloud without being absorbed and steeper the change in the refractive index, the slower will be the velocity of the light. Similarly, the gain-assisted super-luminal light propagation has also been demonstrated in yet another experiment [17]. The absorption curve in (Fig. 1a) is having a very sharp dip, *viz.*, the linewidth is considerably reduced



**Fig. 2.** (a) The absorption spectrum for the parameters  $\Omega_0/\gamma = 0.3$ ,  $N = 5$ ,  $\Phi = 0$ ,  $\delta/\gamma = 0.0$  and  $|M| = \sqrt{N(N+1)}$ . Curves A, B, and C are for  $\Delta/\gamma = 0.3, 0.56$ , and  $0.7$  respectively. (b) The dispersion spectrum for the data as in (a).

and it is showing some gain at the central frequency and at the same time there is very sharp variation in the refractive index (Fig. 1b). So, the situation is very much similar to what has been observed in references [16,17]. The light in a medium of refractive index  $\mu(\omega)$  propagates with a group velocity  $v_{gr} = c/\mu_{gr}$ . The group velocity index is defined [18] by  $\mu_{gr} = \mu(\omega) + \omega d\mu(\omega)/d\omega$ . In the experiment of [16], the group velocity index greatly enhanced due to sharp variation of  $\mu(\omega)$  in normal dispersion region, while in the experiment of [17],  $d\mu(\omega)/d\omega$  is negative but large in magnitude in anomalous dispersion region and so the  $v_{gr}$  can exceed  $c$  and even becomes negative. The actual magnitude of  $v_{gr}$  is very sensitive on the system parameters and on the wavelength of transition. We do not give that here but emphasize that this system is also exhibiting the same phenomenon as mentioned in [16,17]. The difference in our study and that of references [16,17] lies in the mechanism of transparency. In our case the transparency is induced due to the redistribution of the noise around the atom as there is squeezing present in the system. When there is a squeezed reservoir in the system the noise distribution is no longer isotropic in nature and it is changed to an ellipsoid whose orientation is decided by the phase. So, we are getting this unusual behaviour as a result of phase sensitive noise ellipsoid. On the contrary



**Fig. 3.** (a) The absorption spectrum for the parameters  $\Omega_0/\gamma = 0.3$ ,  $N = 5$ ,  $\Delta/\gamma = 0.0$ ,  $\delta/\gamma = 0.2$  and  $|M| = \sqrt{N(N+1)}$ . Curves A, B, C, and D are for  $\Phi = 0, \pi, \pi/2, -\pi/2$  respectively. (b) The dispersion spectrum for the data as in (a).

the mechanism reported in references [16,17] is due to the quantum interference effect.

In Figure 2, we keep non-zero atomic detuning ( $\Delta$ ) to re-examine the above effect. The other parameters are  $N = 5$ ,  $\Phi = 0$ ,  $\Omega_0/\gamma = 0.3$ ,  $\delta/\gamma = 0$  and  $|M| = \sqrt{N(N+1)}$ . Curves A, B, C in Figures 2a and 2b are for  $\Delta/\gamma = 0.3, 0.56$  and  $0.7$  respectively. For the values of parameters selected here the curve A shows gain, curve B shows zero absorption and curve C shows dip in the absorption near  $D = 0$  in Figure 2a. There is asymmetry in the absorption spectrum which increases with increase in  $\Delta$  from A to C. Corresponding dispersion curves A, B and C are displayed in Figure 2b. These curves also show asymmetry which is increasing with  $\Delta/\gamma$ . Also, these dispersion curves show anomalous dispersion where the variation in the refractive index near  $D = 0$  is very sharp for curve A. This sharpness of variation reduce down for the curves B and C. This implies that with finite atomic detuning the enhancement of group velocity index goes down. Next, we study the effect of squeezed vacuum detuning on the absorption dispersion spectra. In Figures 3a and 3b we keep the parameters  $N = 5$ ,  $\Omega_0/\gamma = 0.3$ ,  $\Delta/\gamma = 0.0$ ,  $\delta/\gamma = -0.3$ , and  $|M| = \sqrt{N(N+1)}$ . In Figures 3a and 3b, curves A, B, C, and D are for  $\Phi = 0, \pi, \pi/2$ , and  $-\pi/2$

respectively. With finite atomic squeezed vacuum detuning we loss the effect of negative peak in the absorption spectrum and there is no anomalous spectrum. However, there is phase sensitivity in both absorption and dispersion spectra at the harmonics of  $\delta$ . The unusual behavior of the group refractive index discussed above is because of the atomic medium can have larger nonlinear susceptibilities. The experimental viability of viewing these effects stand good chance in view of recent experiment [19] where a single pulse of light has been used to see the enhancement of group refractive index in ruby crystal. Also, the same group has discussed the electromagnetically induced transparency in a two-level atom and enhancement of susceptibility due to control field [20]. In the work of reference [20] a driving field and a probe field has been used quite similar to the present work. So, alternative method is by sending the weak probe beam in the system to observe this effect of refractive index enhancement.

### 3 The resonance fluorescence spectrum

The steady state fluorescence spectrum is given by [21]

$$S(D_1) = 2\Gamma_0 \text{Re} : \text{Lim}_{t \rightarrow \infty} \int_0^t dt' e^{-\Gamma_0(t-t')} \times \int_0^{t-t'} dt'' e^{(\frac{\Gamma_0}{2} - iD_1)t''} \langle S_+(t' + t'') S_-(t'') \rangle, \quad (10)$$

where  $\Gamma_0$  is the Fabry-Perot interferometer width and  $D_1 = \omega - \omega_L$  is the detuning of the interferometer line centre from the driving field frequency  $\omega_L$ . The steady state spectrum in (10) under weak field excitation (to  $O(\Omega_0^2)$ ) can be expressed as

$$S(D_1) = 2\text{Re} \left[ C_z \left( \frac{\Gamma_0}{2} + i(D_1 - \delta) \right) F_z(\infty) + C_- \left( \frac{\Gamma_0}{2} + i(D_1 - \delta) \right) F_-(\infty) \right], \quad (11)$$

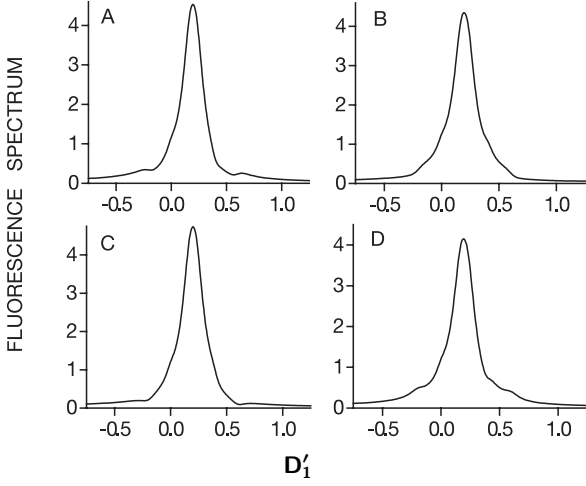
in which  $s$  is the Laplace variable and

$$C_z(s) = \frac{B_+(s)}{\rho(s)}, \quad C_-(s) = \frac{-2i\Omega_0}{\rho(s)} (B_0(s) - \frac{1}{2}B_z(s)), \quad (12)$$

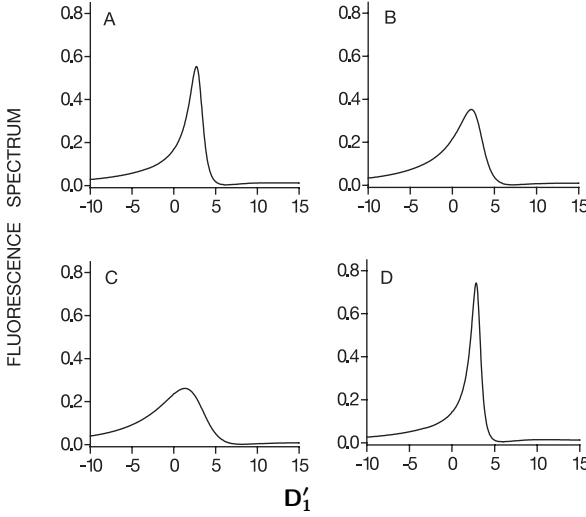
where  $\rho(s)$ ,  $B_z(s)$ ,  $B_+(s)$  are as defined in equations (6, 7) above. Further we have

$$B_0(s) = (s + \lambda_1^*)a_0(s) + \gamma M a_0^*(s), \quad a_0(s) = \frac{1}{2(1+2N)} \left( \frac{1}{s+2\Gamma+i\delta} - \frac{1}{s+i\delta} \right). \quad (13)$$

We plot the resonance fluorescence (RF) spectrum as a function of  $D'_1 = (\omega - \omega_L)/\gamma$  for a two-level atom under the weak field excitation in Figures 4 and 5. In Figure 4 what we have shown essentially is the phase



**Fig. 4.** The resonance fluorescence spectrum (in arbitrary unit) as a function of  $D'_1 = (\omega - \omega_L)/\gamma$  (dimensionless) for a driven two-level atom ( $\Omega_0/\gamma = 0.3$ ) damped by an off-resonant broadband squeezed vacuum with parameters  $N = 5, \Delta/\gamma = 0, \delta/\gamma = 0.2, |M| = \sqrt{N(N+1)}$ . Here curves A, B, C, and D are for the phases  $\Phi = 0, \pi, \pi/2,$  and  $-\pi/2$  respectively.



**Fig. 5.** The resonance fluorescence spectrum (in arbitrary unit) as a function of  $D'_1 = (\omega - \omega_L)/\gamma$  (dimensionless) for a driven two-level atom with  $\Omega_0/\gamma = 0.3$  damped by an off-resonant broadband squeezed vacuum ( $N = 5, \phi = 0$ ),  $|M| = \sqrt{N(N+1)}$ . Here curves A, B, C, and D are for  $(\delta/\gamma = 3, \Delta/\gamma = 0.0), (\delta/\gamma = 3, \Delta/\gamma = 1.0), (\delta/\gamma = 3, \Delta/\gamma = 2.0),$  and  $(\delta/\gamma = 3, \Delta/\gamma = -1.0)$  respectively.

sensitivity of the RF spectrum for the case under consideration. The phase here means the relative phase of the squeezed vacuum with respect to the driving field. The parameters kept in the Figure 4 are as follows:  $N = 5, \Omega_0/\gamma = 0.3, \Delta/\gamma = 0, \delta/\gamma = 0.2$  and  $|M| = \sqrt{N(N+1)}$ . There is a shift in the main peak towards the positive  $D'_1$ -side due to non-zero  $\delta$ , and there are variations observed at the place where  $D'_1 = 2\delta$ . The phase sensitivity is dis-

played for the main peak, however it is more evident at  $D'_1 = 2\delta$  where the four curves plotted for  $\Phi = 0, \pi, \pi/2,$  and  $-\pi/2$  clearly segregate under weak field excitation in the off-resonant squeezed bath. Similarly, there is difference in the peak heights (height of the main peak in C is the largest followed by A, B, and D) as well as slightly different behavior at  $D'_1 = -\delta, 3\delta$  also. The results obtained in [13] are averaged over a long time and hence insensitive to the phase, but we do not make any such assumption here. We do not make any other approximation here except that the driving field is very small and hence perturbation techniques are applicable. So, within that approximation these results show the phase sensitivity.

In Figure 5, on the other hand we bring out the dependence of the RF spectrum on the detuning of the squeezed vacuum ( $\delta$ ) and the atomic detuning ( $\Delta$ ) together. We have kept  $N = 5, \Phi = 0, \Omega_0/\gamma = 0.3$  and  $|M| = \sqrt{N(N+1)}$ . Here curve A is for  $\delta/\gamma = 3, \Delta/\gamma = 0$  and we observe a hole burning at  $D'_1 = 2\delta$ . In curve B, we have kept finite  $\Delta$  such that  $\delta/\gamma = 3, \Delta/\gamma = 1$  and now the hole burning is diffused and shifted towards  $D'_1 = 2\delta + \Delta$ . Further change of  $\Delta$ , *i.e.*, ( $\Delta/\gamma = 2$ , curve C) confirms the position of hole burning as mentioned above. In curve D ( $\delta/\gamma = 3, \Delta/\gamma = -1.0$ ) the hole burning is shifted towards  $D'_1 = 0$  side and the magnitude of the peak increases and the width decreases. So, essentially with this study we find that the nature of the RF spectrum is quite sensitive to the combination values of the atomic and the squeezed vacuum detunings ( $\Delta, \delta$ ).

## 4 Summary

In this work we have examined the absorption-dispersion spectra as well as the resonance fluorescence spectrum of a two-level atom driven by a weak laser field embedded in an off-resonant broadband squeezed bath. The profiles of both the absorption as well as the dispersion spectrum is found to be very sensitive to the atomic detuning as well as the squeezed vacuum detuning and its phase. The presence of the correlated two-photon nonlinear process due to detuned squeezed vacuum is very much evident in the profiles of the absorption-dispersion spectra. We have also shown the sensitivity of the fluorescence spectrum on the relative phase of the driving field and the squeezed vacuum. The effect of the phase is prominent near the peak as well as near the harmonics of the squeezed vacuum detunings. We have shown a possibility of observing enhanced group velocity effect in our system. That is steep change in the refractive index accompanied with almost zero absorption or gain induced by the squeezed vacuum. Such processes of enhanced group velocity have a link to the coherent optical information storage in atomic media and quantum computers as well as realizing non-linear optical effects (useful for making ultra sensitive switches) at very low photon densities [16]. We have also brought out the sensitive dependence of the fluorescence spectrum profile on the atomic as well as the squeezed vacuum detunings.

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## References

1. L.A. Wu, H.J. Kimble, J.L. Hall, H. Wu, Phys. Rev. Lett. **57**, 2520 (1986); R.E. Slusher, L.W. Hollberg, B. Yurke, J.C. Mertz, J.F. Valley, Phys. Rev. Lett. **55**, 2520 (1985); R.M. Shelby, M.D. Levenson, S.H. Perlmutter, R.G. DeVoe, D.F. Walls, Phys. Rev. Lett. **57**, 691 (1986); M.W. Maeda, P. Kumar, M.J. Shapiro, Opt. Lett. **3**, 161 (1986); S. Machida, Y. Yamamoto, Y. Itago, Phys. Rev. Lett. **58**, 1000 (1987); N.P. Georgiades, E.S. Polzik, K. Edamatsu, H.J. Kimble, A.S. Parkins, Phys. Rev. Lett. **75**, 3426 (1995); N.P. Georgiades, E.S. Polzik, H.J. Kimble, Phys. Rev. **55**, R1605 (1997); Q.A. Turchette, N.Ph. Georgiades, C.J. Hood, H.J. Kimble, A.S. Parkins, Phys. Rev. A **58**, 4056 (1998); for a recent review see B.J. Dalton, Z. Ficek, S. Swain, J. Mod. Opt. **46**, 379 (1999)
2. L. Mandel, E. Wolf, *Optical coherence and Quantum Optics* (Cambridge Univ. Press., Cambridge, UK, 1995), Ch. 21 and references therein
3. C.W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986)
4. H.J. Carmichael, A.S. Lane, D.F. Walls, Phys. Rev. Lett. **58**, 2539 (1987)
5. H. Ritsch, P. Zoller, Opt. Commun. **64**, 523 (1987); Phys. Rev. A **38**, 4657 (1988)
6. S.S. Hassan, H.A. Batarfi, S.K. Ng, M.R. Wahiddin, J. Phys. B **30**, L777 (1997); see also, K.T. Lim, S.S. Hassan, M.R. Muhamad, M.R.B. Wahiddin, Quant. Semiclass. Opt. **9**, L23 (1997)
7. Z. Ficek, W.S. Smyth, S. Swain, Opt. Commun. **110**, 555, (1994); Phys. Rev. A **52**, 4126, (1995)
8. M. Sargent III, Phys. Rep. **43**, 233 (1978); R.W. Boyd, M.G. Raymer, P. Natum, D.J. Harter, Phys. Rev. A **24**, 411 (1981)
9. M.O. Scully, Phys. Rev. Lett. **67**, 1855 (1991)
10. M.O. Scully, M. Fleischhauer, Phys. Rev. Lett. **69**, 1360 (1992); M. Fleischhauer, C.H. Keitel, M.O. Scully, S. Chang, B.T. Ulrich, S.Y. Zhu, Phys. Rev. A **46**, 1468 (1992); M. Fleischhauer, C.H. Keitel, M.O. Scully, C. Su, Opt. Commun. **87**, 109 (1992); M.O. Scully, S.Y. Zhu, Opt. Commun. **87**, 134 (1992); also see, M.O. Scully, M.S. Zubairy, *Quantum Optics* (Cambridge Univ. Press, Cambridge, UK, 1997); S. Alam, *Lasers without inversion and Electromagnetically induced transparency* (SPIE Optical Engineering Press, Bellingham, USA, 1999)
11. A.-S. Zibrov, M.D. Lukin, L. Hollberg, D.E. Nikonov, M.O. Scully, H.G. Robinson, V.L. Velichansky, Phys. Rev. Lett. **76**, 3935 (1996)
12. A. Joshi, S.S. Hassan, J. Phys. B **30**, L557 (1997); S.S. Hassan, A. Joshi, O.M. Frege, Nonlin. Opt. Quant. Opt.: Conc. Mod. Opt. **30**, 23 (2003)
13. Z. Ficek, B.C. Sanders, J. Phys. B **27**, 809 (1994)
14. S.S. Hassan, O.M. Frege, N. Nayak, J. Opt. Soc. Am. B **12**, 1177 (1995); S.S. Hassan, M.R. Wahiddin, R. Saunders, R.K. Bullough, Physica A **215**, 556 (1995); **219**, 482 (1995)
15. R. Kubo, J. Phys. Soc. Jap. **12**, 570 (1957); B.R. Mollow, Phys. Rev. A **5**, 2217 (1972)
16. L.V. Hau, S.E. Harris, Z. Dutton, C.H. Behroozi, Nature **397**, 594 (1999); L.V. Hau, Sci. Am. **285**, 52 (2001) and references therein
17. L.J. Wang, A. Kuzumich, A. Dogariu, Nature **406**, 277 (2000)
18. J.D. Jackson, *Classical Electrodynamics* (Wiley, 1975), Sect. 7.8
19. M.S. Bigelow, N.N. Lepeshkin, R.W. Boyd, Phys. Rev. Lett. **90**, 113903 (2003)
20. R.S. Bennink, R.W. Boyd, C.R. Stroud Jr, V. Wong, Phys. Rev. A **63**, 033804 (2001)
21. J.H. Eberly, C.V. Kunasz, K. Wodkiewicz, J. Phys. B **13**, 217 (1980)